

Report on the Current Status of the Standard Cosmological Model

Prepared for rapid review and criticism by the
participants of the December 2010 workshop:

Experimental and Theoretical Challenges to Probing Dark Energy

[France-Stanford Center for Interdisciplinary Studies](#)

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An official response by U.S. workshop participants is requested.

Preamble

Although this communication is being more widely disseminated, its official recipients are the participants in the December 2010 workshop, “Experimental and Theoretical Challenges to Probing Dark Energy” listed in Appendix I. *Those participants who are U.S. citizens are requested to provide a collective official response to this communication pursuant to the bylaws of the American Physical Society and to federal statutes of the United States regulating the activities of its employees as well as individuals and their institutions receiving federal funding for scientific research and development activities.* These APS bylaws, federal statutes and consequences for non-compliance were established to ensure ethical behavior by scientific professionals, putting objective truth (and so the interest of the world scientific community and all United States citizens) ahead of any possible conflicting personal interests. To ensure that the requested response is readily accessible to the scientific community and the public, it shall be posted in the *Astrophysics* (astro-ph) section of the arXiv.org website in accord with the standard protocol for communicating similar academic information. It is anticipated that French citizens will request a similar response from their participating compatriots, which may or may not be independent of an American response. The deadline for the requested official American response is 1 March 2011 (i.e., within three months).

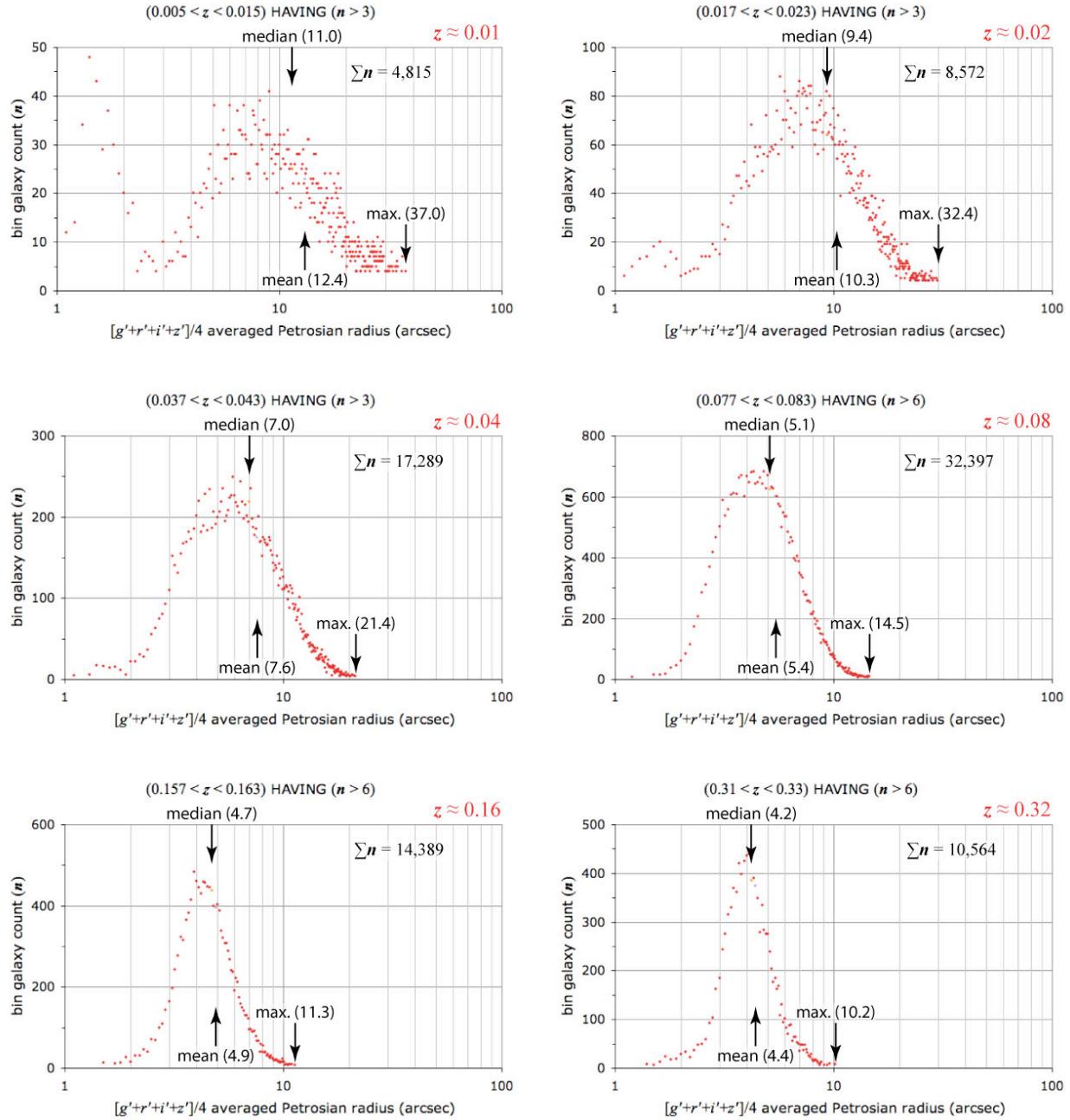
Introduction

As we approach the close of 2010, the state of physical science is that of a publicly unresolved crisis of unprecedented scope in modern history. Among the most critical symptoms of this crisis in science is the interpretation of various astrophysical observations as an inexplicable accelerating cosmic expansion. The December 2010 workshop, “Experimental and Theoretical Challenges to Probing Dark Energy,” at Stanford University, sponsored by the France-Stanford Center for Interdisciplinary Studies, brings together recognized leaders with the goal of resolving this crisis.¹

The pervasive assumption in academia that the basic tenets of the current standard cosmological model require no modification has prevented resolution of this crisis. Historical precedent teaches that similar major scientific crises are resolved by a new perspective leading to profound changes in the interpretation of empirical observables.

Theoretical physics that is of value yields an accurate mathematical model of observable and testable empirical phenomena. Furthermore, the quantitative model must be in support of a comprehensive and logically consistent qualitative model that rests on established first principles that have been verified in the laboratory (e.g., the local invariance of the speed of light in vacuum). An additional requirement is an objective simplicity, which implies a lack of free parameters that can be manipulated *a posteriori* to alter theoretical predictions so that they fit known empirical observations.

Three new predictive equations are presented herein that meet all of these requirements, providing correlated fits of unprecedented accuracy to a variety of accurate astrophysical observations while removing the need for highly speculative assumptions. It is important to understand that all three equations originate from the same theoretical foundation, which is the new concept of cosmic relativistic temporal geometry in a finite boundaryless spacetime Universe. Accordingly, these three *a priori* equations have no independent existence; each equation making a distinct prediction implies the other two equations. No free parameters are incorporated in these equations, so there is no possibility of subjectively fitting the predictive models to their respective empirical datasets.



Sloan Digital Sky Survey (SDSS) apparent galaxy radius data

Figure 1 | Six SDSS* redshift bins, each with a continuous spectrum of constituent galaxy sizes. Redshift bins are $z = 0.01; 0.02; 0.04; 0.08; 0.16; 0.32$, doubling the redshift for each consecutive bin. Galaxy size plotted on the horizontal axis within each redshift bin is the average of the four individual Petrosian radius measurements for the SDSS (g' , r' , i' , and z') bandpass filters. A Petrosian radius uses the radial light curve to account for variations in galaxy morphology (i.e., shape) and orientation. The SDSS data implies that the largest galaxies are typically not more than about three times the diameter of average-sized galaxies and not more than about ten times the size of normal small galaxies. Some of the smallest objects at lowest redshift are likely to be misidentified double stars. The smallest objects at higher redshift may reflect unusually bright active galactic nuclei, which dominate the radiation output of the host galaxy. Bin population minimums were required to eliminate anomalous unphysical entries in the database, which were checked manually. *The statistical median galaxy radius and mean galaxy radius trend closely with the observed maximum galaxy radius as a function of redshift.*

Theta-z relationship

The *theta-z* relationship predicts the apparent angular size (θ) of an astrophysical object of uniform size (i.e., a “standard rod”) with cosmological redshift (z). The only distinct objects large enough to be measured over the required range of cosmological distance are galaxies, which are known to vary in diameter regardless of uniformity in other individual identifying characteristics. However, analysis of galaxy populations in the Sloan Digital Sky Survey (SDSS) at nearly the same redshift distance ($z \pm 0.003$) over a range of redshift shows a consistent statistical variation in galaxy size (Fig. 1) that extends over about one order of magnitude (i.e., a factor of $\sim 10^1$). Accordingly, the average galaxy size for an SDSS redshift bin with a suitably large population (i.e., $\sim 10k$ galaxies randomly distributed over about one quarter of the sky) provides a consistent *statistical* astrophysical standard rod over this observed range in redshift. In the comparative graph in Fig. 2, a systematic effect with redshift on the sizes of all galaxies, which would produce a systematic effect on the average galaxy size within this range of redshift ($0.02 \leq z \leq 0.1$) is ruled out.

According to the standard cosmological model, the observed redshift of galaxies is due to a general expansion of space between the galaxies causing a general recessional motion of the galaxies relative to one another. This interpretation implies the so-called ‘Hubble law,’ whereby the recession velocity of a galaxy is proportional to its distance in accord with the ‘Hubble constant’ (H_0). Uniformly expanding space implies a linear relationship between galaxy separation distance (d) and recession velocity (v).

$$v = H_0 d \quad (1)$$

For cosmological redshifts in the range ($0.02 \leq z \leq 0.1$), the measured redshift (z) would be the corresponding recession velocity divided by the speed of light (c), which would then imply that distance to a galaxy is directly proportional to its measured redshift.

$$z = \frac{v}{c} \rightarrow v = cz \rightarrow cz = H_0 d \rightarrow d \propto z \quad (2)$$

The angle (θ) subtended by an object at some distance is just the ratio of the physical diameter of the object to the complete circumference of a circle at that distance, which represents an angle of 360 degrees or 2π radians (i.e., $\sim 6.283 \times$ the circle’s radius). Assuming a Euclidean space, because the circumference of a circle ($2\pi r$) is directly proportional to its radius, if one doubles the radius (i.e., the distance, d) to an object, the object’s apparent size (i.e., the angular portion θ of the complete circle 2π taken up by the object’s fixed length, D) decreases by a factor of two. It is important to consider that a Euclidean space is not a logical assumption in the context of cosmological distances.

$$\theta = \frac{D}{d} \quad (3)$$

Substituting an unknown galaxy distance (d) with an accurately-measured redshift (z), it is readily apparent from equation (4) that for an expanding universe governed by the ‘Hubble law,’ the measured average galaxy size at a given redshift must be inversely proportional to the measured cosmological redshift; in an expanding universe, at half the redshift ($z \sim 0.1$) one anticipates the apparent angular size of a standard rod to double.

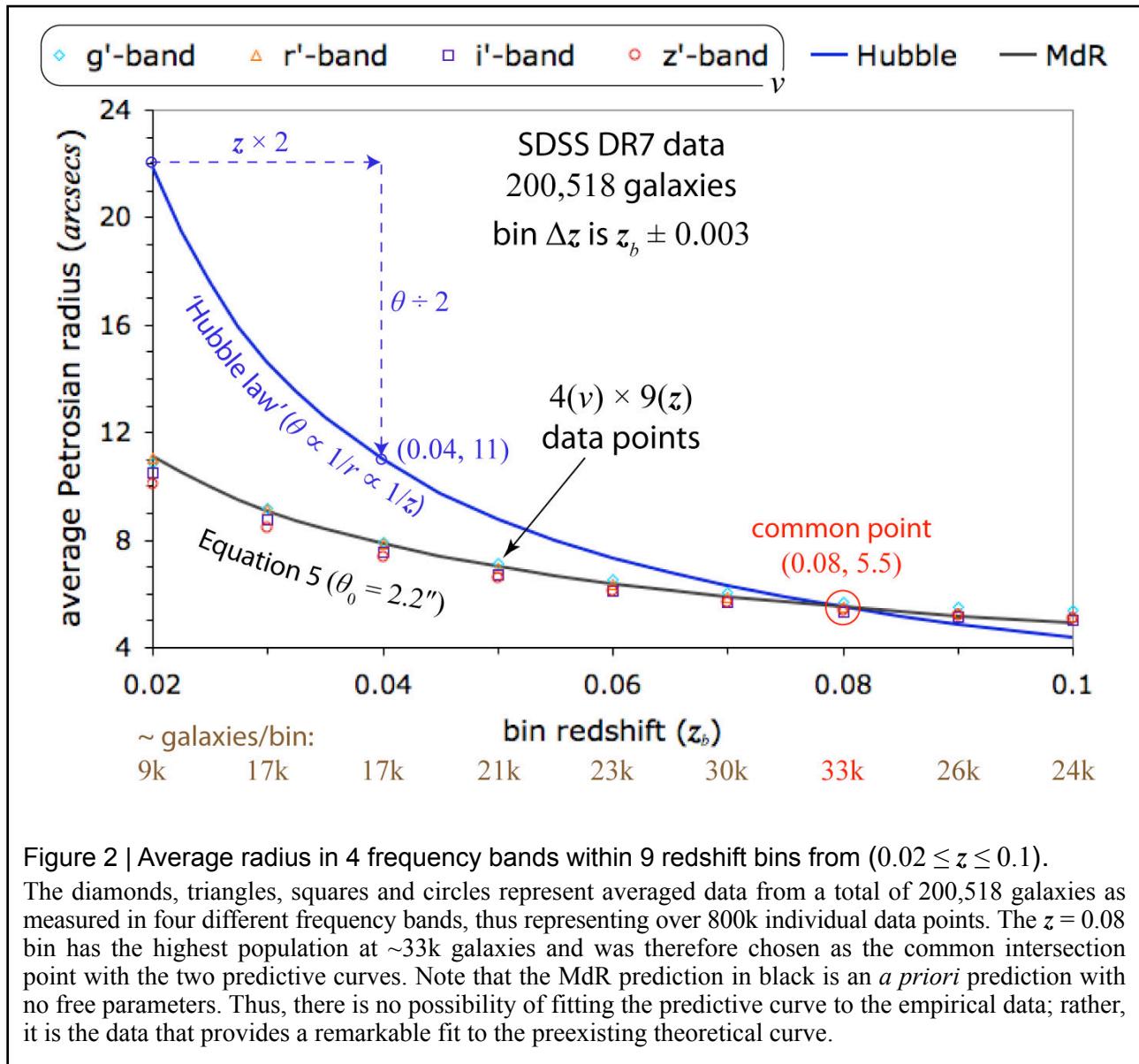
$$\theta \propto \frac{D}{z} \quad (4)$$

This relationship is plotted as the blue curve in Fig. 2 where it is compared to SDSS empirical data. The common point (0.08, 5.5'') between the data and the predictive curve was selected for having the largest galaxy population of the nine empirical redshift bins and therefore the most accurate bin average for the measured Petrosian radius.

In contrast to the forgoing idea, a synthesis of ideas originating with Hermann Minkowski, Willem de Sitter and Bernhard Riemann (i.e., the “MdR cosmological model”) yields a competing predictive equation based exclusively on the axioms of relativity.²

$$\theta(z) = \theta_0 (z+1)^{-1} \left(1 - \frac{1}{(z+1)^2} \right)^{-\frac{1}{2}} \quad [z > 0] \quad (5)$$

This relationship is plotted as the black curve in Fig. 2, having the same common point with the data (0.08, 5.5'') as the ‘Hubble law’ curve in blue.



The MdR model recognizes that just as time is a strictly local (i.e., “proper”) relativistic phenomenon in the context of relative motion, it is similarly a strictly local relativistic phenomenon in the context of cosmological distance. The greater the distance between ideal clocks, the greater the relativistic time dilation between them, up to the boundary of the cosmological redshift horizon ($z = \infty$) at a distance of about 15 billion light years. This time dilation effect is non-linear with cosmological distance and it is unrelated to relative motion. The horizon distance estimate is based on intermediate measurements. No absolute timescale applicable to the entire Universe (i.e., ‘conformal time’) exists. Equation (5) is a purely theoretical *a priori* prediction arising from first principles with no free parameters to allow for curve fitting; the variable θ_0 is a simple scaling parameter. For example, given that the observed average apparent galactic diameter at $z = 0.08$ is 5.5 arcseconds, the value of the constant θ_0 in this equation, which is valid for all redshifts, is set to 2.2 arcseconds. If some other standard rod of a different intrinsic size existed and was being modeled, θ_0 would be set to some other value.

The SDSS data shown implies a catastrophic failure of the ‘Hubble law.’ Moreover, the statistical averaging of the SDSS Petrosian radius measurements is so accurate that an expected selection effect is evident in reference to the MdR prediction. At high redshift, one expects smaller galaxies to drop out of the sample. At low redshift, the largest and brightest galaxies are not included in the survey by choice. Accordingly, one sees a transition of the observational data from slightly below the predicted curve at low redshift to slightly above the curve at high redshift. This strongly suggests that the Eq. 5 predictive curve correlates to theta-z measurements for an ideal astrophysical standard rod, which is closely approximated by the statistically averaged SDSS measurements.

There are those with a vested interest in the *status quo* who might argue *ad absurdum* that Fig. 2 does not imply a failure of the ‘Hubble law,’ nor a confirmation of the competing prediction. Consider that such an argument asserts that the fit of the empirical data to the Eq. 5 prediction is a random accident with no physical meaning in spite of this equation’s *a priori* derivation from first principles. Moreover, one would have to explain why the empirical data does not fit the canonical prediction and also develop a convincing argument to justify the *a posteriori* fitting of the empirical data to the blue ‘Hubble law’ curve. Recall that Fig. 1 shows the detailed distribution of measured galaxy sizes for SDSS redshifts bins within the range shown in Fig. 2.

An empirical failure of the ‘Hubble law’ necessarily implies that the canonical idea that the Universe is expanding is incorrect. Cosmic expansion logically requires a linear redshift-distance relationship; ignoring any minor variation arising from an assumed acceleration, uniformly expanding space implies that at twice the distance, galaxies are receding at twice the speed. If the empirical evidence does not support a linear redshift-distance relationship, the popular ‘expanding universe’ paradigm and all of the many secondary assumptions associated with this idea must be abandoned and subsequently explained in total by a new comprehensive theory. The MdR model achieves this.

In addition to the apparent fact that the ‘Hubble law’ is incorrect, the SDSS theta-z data accurately modeled by Eq. 5 implies that galaxies at intermediate redshift ($z < 1$) are considerably closer than modeled by the canonical linear redshift-distance relationship. *If galaxies at increasing redshift are actually considerably closer relative to nearby galaxies than predicted by the standard model, then they will dim less rapidly with increasing redshift (i.e., they will appear brighter) than predicted by the standard model.*

Redshift-magnitude relationship

The astronomical magnitude scale used to measure the apparent brightness of an astrophysical object is a *reverse logarithmic* scale where five magnitudes or “*mags*” equates to a factor of 100 in brightness. Then, a 1-watt bulb has an apparent magnitude that is 5 units *greater* than a 100-watt bulb at the same distance. Apparent luminosity is inversely related to the area of a sphere ($4\pi r^2$) over which a central isotropic radiation source distributes its photons. At ten times the distance (i.e., radius), a concentric sphere has $10^2 = 100 \times$ the surface area, which implies that the apparent luminosity of the central source decreases by a factor of 100 (i.e., its apparent magnitude *increases* by 5 *mags*).

A “standard candle” is a common astrophysical object that has very nearly the same intrinsic luminosity. Because the ‘Hubble law’ implies that an object at ten times the redshift ($z \sim 0.1$) is ten times farther away, the corresponding redshift-magnitude curve for a standard candle is a straight line showing an increase of 5 *mags* for a tenfold increase in redshift. This relationship is shown as the straight blue line in Fig. 3. At redshifts approaching $z = 1$, the actual canonical redshift-magnitude curve differs only slightly from this simple blue line, exhibiting a slightly increasing slope with increasing redshift.

In contrast to the forgoing idea, the same mathematical model of the Universe that yields the MdR theta-z relationship shown in Fig. 2 as the black curve yields a competing predictive equation for the redshift-magnitude relationship of a standard candle.

$$m(z) = C - 2.512 \log \left(\frac{1}{4\pi [(z+1)^4 - (z+1)^2]} \right) \quad (6)$$

This relationship is plotted as the solid black curve in Fig. 3, adjusted by C to have a common point of (0.02, 14.5) with the data and the ‘Hubble law’ curve. This intersection is the lowest point in the data within the survey magnitude limit for the i' -band.

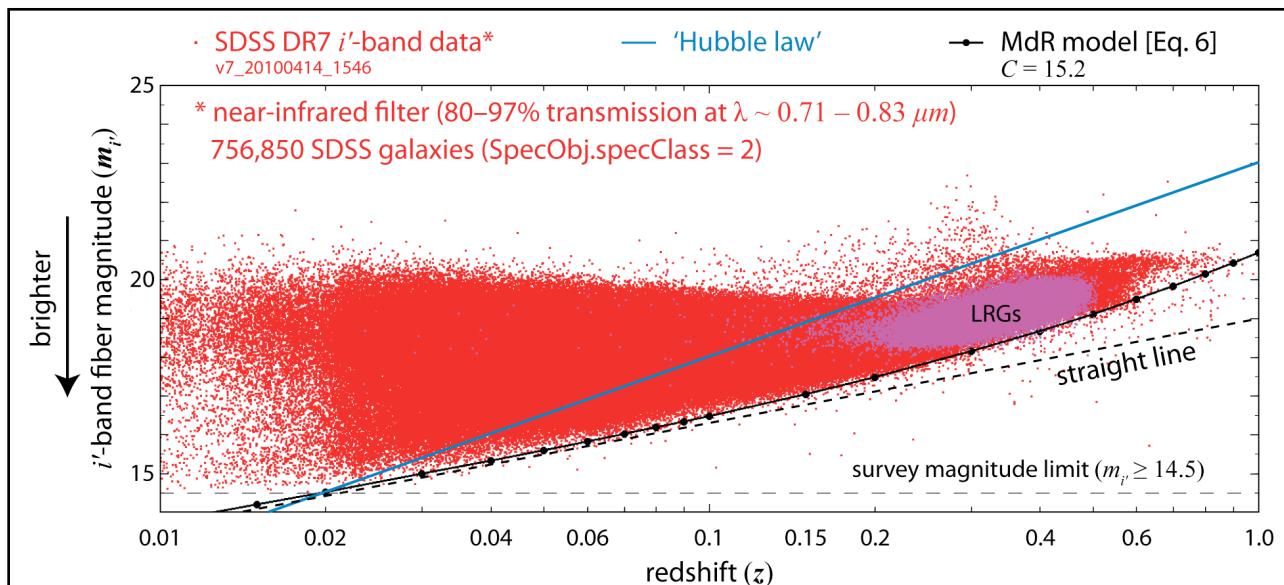


Figure 3 | Direct redshift-magnitude measurements for over 750k galaxies in the i' -band. This data is prior to K-correction, which accounts for changes in emission luminosity with the z -induced wavelength shift. The K-correction at $z = 0.4$ for LRGs is +0.35, which accounts for the gap at low redshift between the data and the MdR curve.³ “LRGs” in violet refers to the survey of Luminous Red Galaxies.

It is conventionally assumed that the SDSS fiber magnitudes, which are unadulterated empirical measurements of the flux within the 3-arcsecond aperture of a spectroscopic fiber, require substantial correction to account for geometric distance effects. Fig. 4, which shows the SDSS g' -band redshift-magnitude data, provides conclusive empirical evidence that this prior assumption was incorrect. The rapid decline in galaxy luminosity with decreasing observed wavelength seen in the 4000-Angstrom (Ca II) “break” region is created primarily by accumulation of absorption lines in metal-rich stellar populations of galaxies. *The typical brightest galaxies making up the base of the Fig. 4 dataset must represent effective standard candles; otherwise, the observed abrupt galaxy luminosity change of about 1.5 mags at $z \approx 0.4$ would not be evident in the data.*

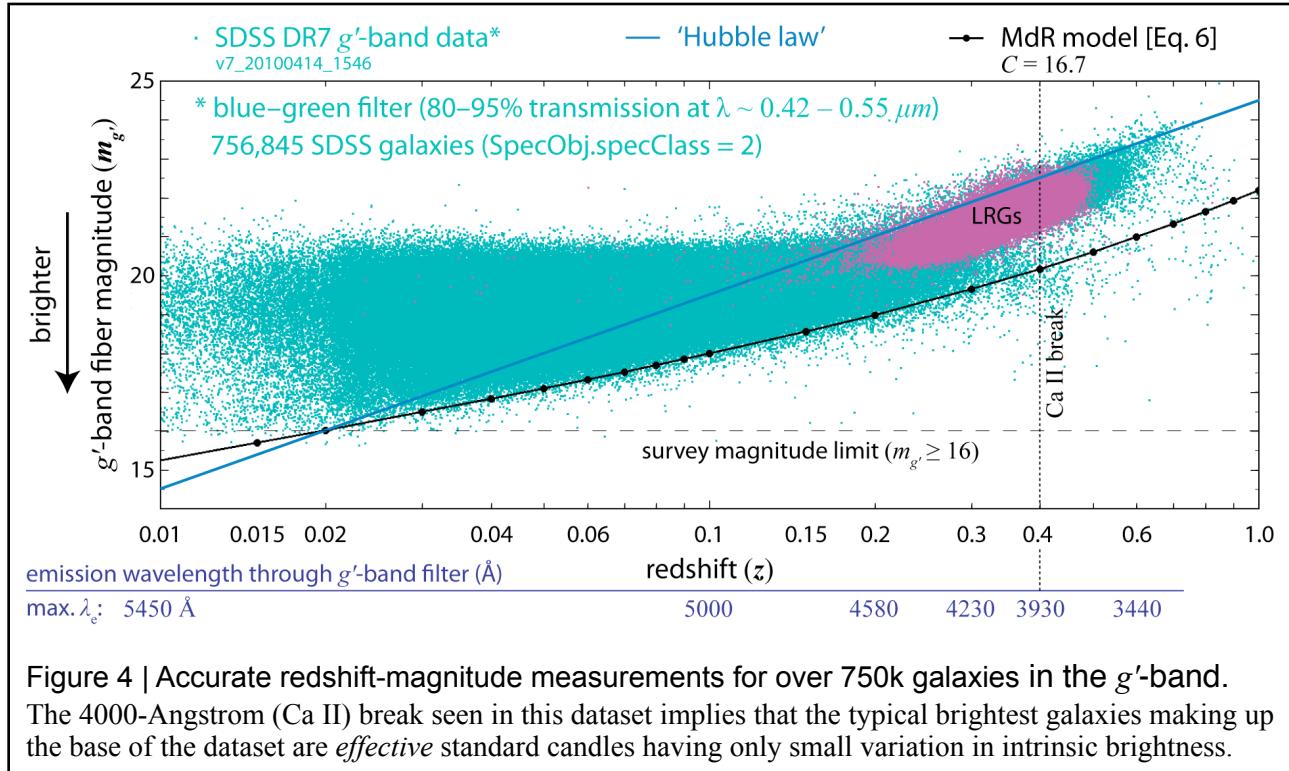


Figure 4 | Accurate redshift-magnitude measurements for over 750k galaxies in the g' -band. The 4000-Angstrom (Ca II) break seen in this dataset implies that the typical brightest galaxies making up the base of the dataset are *effective* standard candles having only small variation in intrinsic brightness.

The SDSS data shown in both Fig. 3 and Fig. 4 again implies a catastrophic failure of the ‘Hubble law.’ According to the canonical blue curve, if one imagines transporting the brightest galaxies seen at redshift 0.03 to redshift 0.3 (allegedly ten times farther away), their apparent luminosity must increase by 5 mags. However, the empirical data shows an increase of just 3 mags for the typical brightest galaxies, which definitely serve as effective standard candles. This is an observed decrease in apparent luminosity more than six times less than the expected decrease required for an expanding universe.

On the other hand, the *a priori* redshift-magnitude prediction of the MdR model provides an essentially perfect fit to the SDSS data, just as was true for the SDSS theta-z data. The SDSS theta-z and redshift-magnitude datasets are independent of one another; that both datasets are perfectly predicted by two inextricable *a priori* mathematical models indicates that the new theory behind these two correlated models is correct.

Although one can already draw certain conclusions from the remarkable accuracy of the aforementioned corresponding MdR predictions, there is a third independent SDSS dataset that extends empirical confirmation of the MdR model to very high redshift.

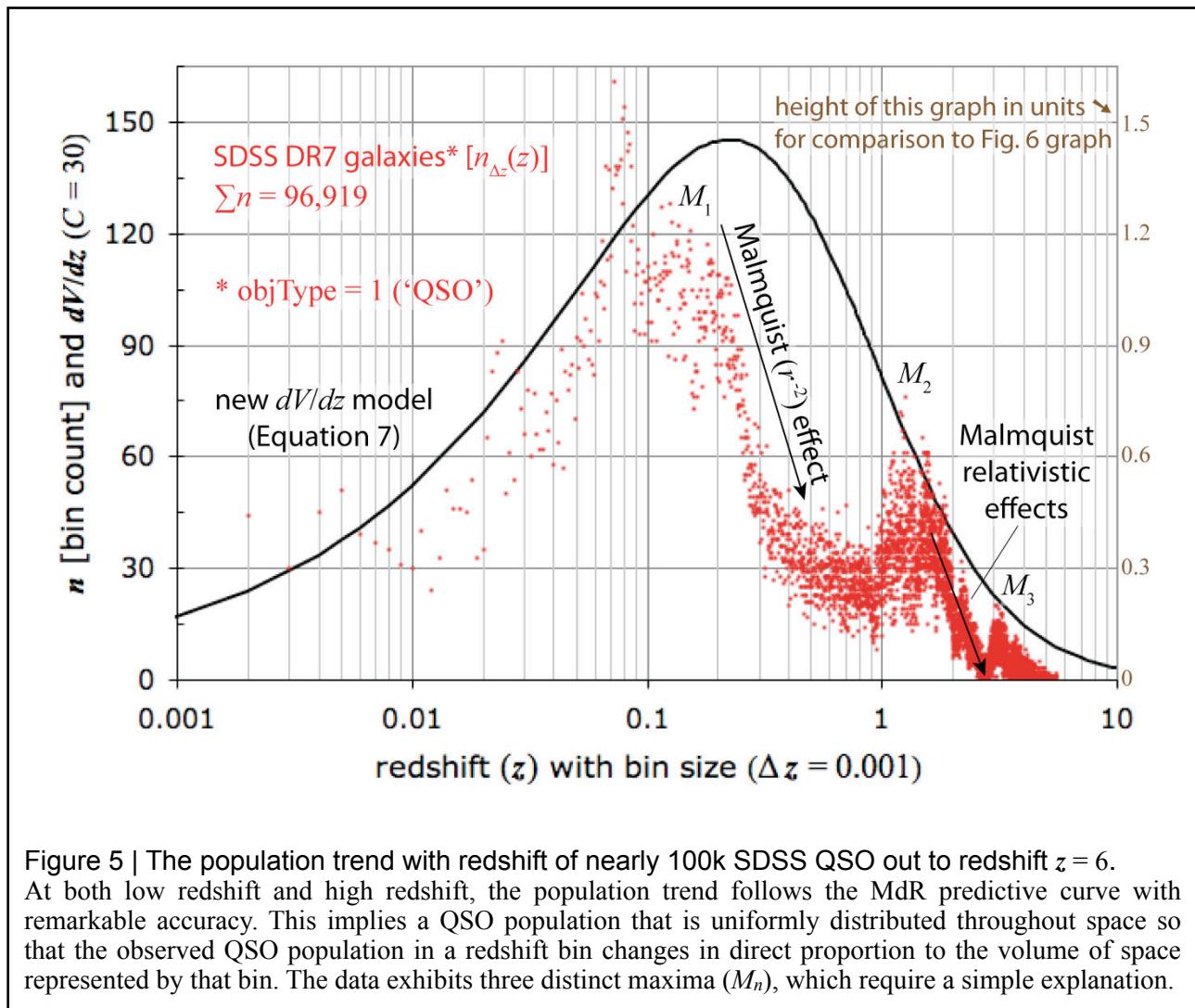
Volume-redshift (dV/dz) relationship

The current standard cosmological model assumes Euclidean space. This assumption, together with the ‘Hubble law,’ implies that the volume of enclosed space grows as the third power of the redshift ($z < 0.2$) in accord with the volume of a sphere ($4/3\pi r^3$). The rate of change of spatial volume with redshift (i.e., the calculus derivative dV/dz) then grows as the second power of the redshift in accord with the surface area of the sphere ($4\pi r^2$) enclosing that space. This relationship is plotted as the blue curve in Fig. 6.

In contrast to the forgoing idea, the MdR cosmological model, which recognizes that cosmic space is the finite boundaryless volumetric ‘surface area’ of a 3-sphere with time as the *local* vertical, yields a competing predictive equation for the dV/dz relationship.

$$\frac{dV}{dz} = \frac{4\pi C}{\sqrt{1-(z+1)^{-2}}} \left(\frac{1}{(z+1)^2} - \frac{1}{(z+1)^4} \right) \quad (7)$$

This relationship is plotted as the black curve in Fig. 5. The arbitrary constant C is a scaling parameter that sets the height of the curve on the vertical axis to match the maximum bin populations of the particular empirical dataset being examined. $C \approx 30$ for SDSS QSOs, but for some other redshift survey, the value of C would typically differ.



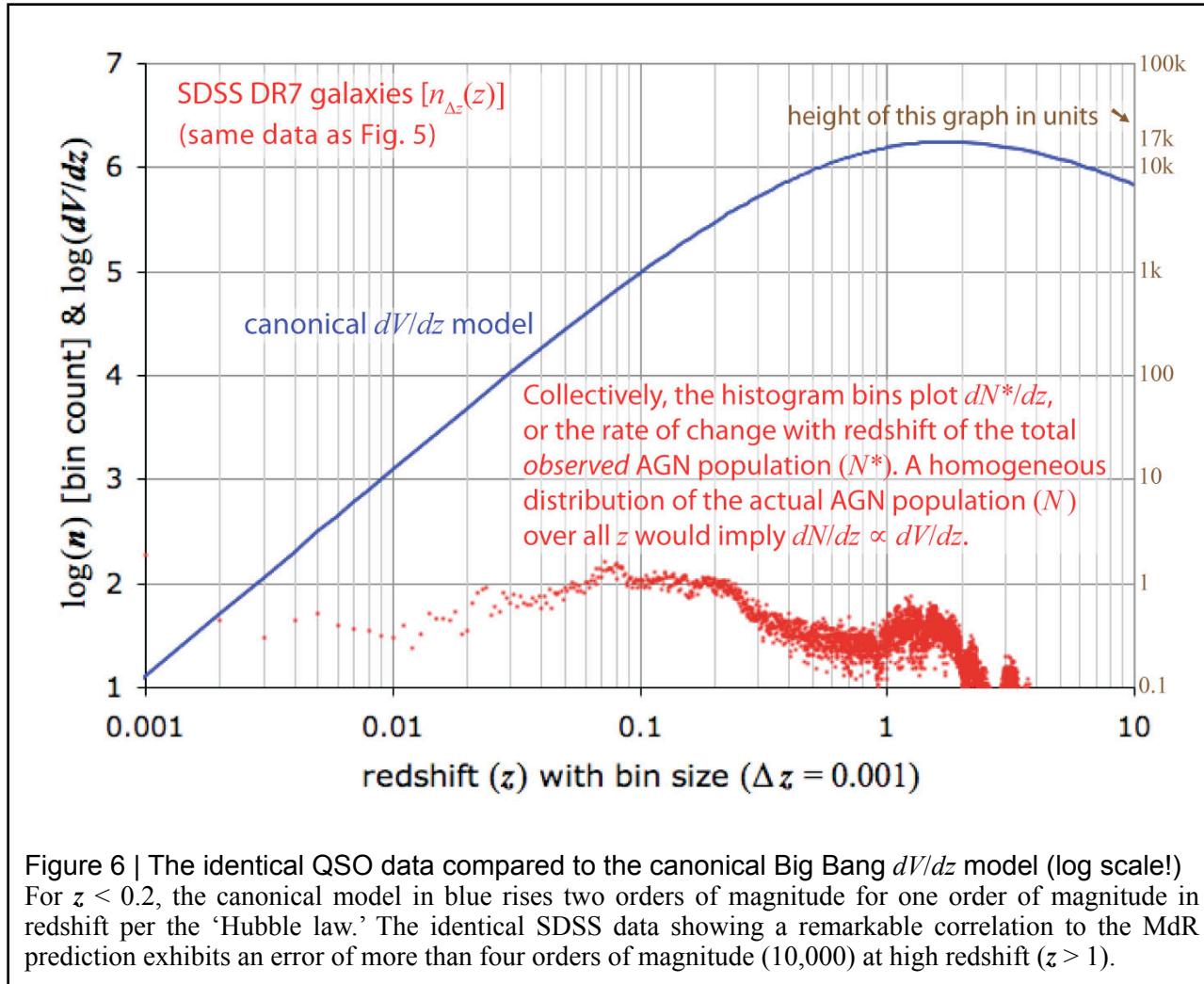


Figure 6 | The identical QSO data compared to the canonical Big Bang dV/dz model (log scale!). For $z < 0.2$, the canonical model in blue rises two orders of magnitude for one order of magnitude in redshift per the ‘Hubble law.’ The identical SDSS data showing a remarkable correlation to the MdR prediction exhibits an error of more than four orders of magnitude (10,000) at high redshift ($z > 1$).

The conventional interpretation of QSO population data assumes an evolutionary process over lookback time whereby a higher percentage of galaxies contained active galactic nuclei (i.e., QSO) in the past than is the case in the local universe. The MdR theory does not associate lookback time with a cosmic evolutionary process relative to a universal cosmic timescale (i.e., ‘conformal time’), which does not exist. Rather, it is evident that every region of the Universe is generally similar to itself and every other region over an arbitrary amount of time as measured by an ideal clock anywhere.

In the MdR cosmology, which is based in part on a correction to the way in which the general theory of relativity treats time, QSO typically represent white holes, which are local mass-energy sources. Cosmic energy conservation is maintained because each such white hole has a corresponding local mass-energy sink (i.e., a black hole) that is located at the cosmic antipode to the white hole. A black hole is not a spacetime singularity, where the laws of physics break down, which cannot and does not exist; rather, a black hole is the energy sink region of an unstable cosmic toroidal spacetime geometry maintained by an energy flow. The space density of QSO, which represent cosmic wormholes, are necessarily and demonstrably uniform throughout the Universe. Wormholes are ubiquitous and transport mass-energy between cosmic antipodes, thereby preventing catastrophic cosmic gravitational collapse over an infinite timescale.

The three distinct maxima in the QSO data ($z \approx 0.1, 1.5$, and 4) are easily interpreted. It is evident that bright active galactic nuclei may be categorized in three distinct groups according to variation in intrinsic luminosity: (1) ordinary, (2) brighter and (3) brightest. Up to about redshift $z = 0.1$, all QSO are visible, so one observes the bin population trend with the modeled increase in bin volume. The M_1 maximum marks the redshift prior to which ordinary QSO begin to drop out of the SDSS sample. By redshift $z = 0.5$ more than 75% of the total QSO population has drop out of the SDSS QSO sample. The M_2 maximum marks the redshift prior to which unusually bright QSO, representing about 10% of the total population, begin to drop out of the SDSS sample. Dimming in this redshift regime is primarily due to relativistic effects $[(z + 1)^{-2}]$; time dilation causes the telescope CCD camera to receive fewer photons per unit time $[(z + 1)^{-1}]$ than the source emission rate and these redshifted photons arrive with reduced energy $[(z + 1)^{-1}]$. The M_3 maximum marks the redshift prior to which the brightest class of QSO, representing less than 1% of the total population, begin to drop out of the SDSS sample as the resolution limit of the SDSS telescope is reached at nearly $z = 6$.

The magnitude in the error associated with the Hubble interpretation of the cosmological redshift can be appreciated by comparing the QSO redshift-population histogram to the canonical dV/dz curve on a linear scale. Given that the height of the Fig. 5 graph, which provides a good fit to both the data and the MdR curve, is about 10 cm, if the canonical dV/dz curve were printed using the same vertical scale, it would peak at a height of nearly one kilometer above the peak of the empirical data! The Big Bang theory is not just wrong; it is evidence of a systemic 20th-century mental pathology affecting science, art and culture that requires general recognition, understanding and remediation.

Conclusion

The failure of the standard cosmological model to fit accurately-measured and reported empirical data and the contrasting remarkable fit of the *a priori* MdR model predictions to the same data confirms beyond any reasonable doubt that the Big Bang theory of cosmic evolution is completely wrong. It is *certain* that the Universe is not expanding.

The predominant opinion that the standard cosmological model is a scientific fact rather than a pathologically flawed theory is largely based on manufactured data. Given an accurate predictive model that matches accurate empirical observations, it is now easy to demonstrate that direct empirical astrophysical observations objectively recorded by instruments were routinely subjectively altered to fit theoretical predictions, which is the antithesis of science. A false reality was artificially created and sold as objective reality.

Obviously, an *accelerating* cosmic expansion caused by ‘dark energy’ does not exist if no cosmic expansion exists. The only “challenge to probing dark energy” is to recognize that any ‘work’ related to ‘dark energy’ is utterly meaningless in the context of science.

All of the graphed data shown in this report can be easily reproduced from the online SDSS SkyServer database. The MdR theory is developed in detail in the online preview of the pending monograph, *On the Geometry of Time in Physics and Cosmology and the Fall of the Canonical Cosmological Model*, available online at JPhysics.org/book.

Failure to respond appropriately to this communication within the requested timeframe will result in a [motion](#) for Congressional subpoena of selected workshop participants. United States Code Title 18, Part I, Chapter 47, [§ 1001](#) and [§ 1031](#) are applicable.

Appendix I : December 2010 France-Stanford Workshop Participants

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Marusa Bradac	University of California, Davis
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Robert Cahn	Lawrence Berkeley National Laboratory
Sean Carroll	California Institute of Technology
Sarah Church	Stanford University
Michael Dine	University of California, Santa Cruz
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Chris Hirata	California Institute of Technology
Henk Hoekstra	Leiden University
Daniel Holz	Los Alamos National Laboratory
Renata Kallosh	Stanford University
Alex Kim	Lawrence Berkeley National Laboratory
Andrei Linde	Stanford University, SLAC National Accelerator Laboratory
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Leonardo Senatore	Stanford University, SLAC National Accelerator Laboratory
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Michael S. Turner	University of Chicago
Anthony Tyson	University of California, Davis
Risa Wechsler	Stanford University, SLAC National Accelerator Laboratory

[†] oraganizing committee

^{*} co-chair

References

- ¹ <http://francestanford.stanford.edu/conferences/darkenergy>
- ² A. F. Mayer, "On the Geometry of Time in Physics and Cosmology and the Fall of the Canonical Cosmological Model," (prepress preview, June 2010); available at <http://JPhysics.org/book>
- ³ I. Chilingarian, A-L Melchior & I. Zolotukhin (2010) arXiv:1002.2360v1, Fig. 3.